Inflection point as a manifestation of tricritical point on the dynamic phase boundary in Ising meanfield dynamics

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Abstract: We studied the dynamical phase transition in kinetic Ising ferromagnets driven by oscillating magnetic field in meanfield approximation. The meanfield differential equation was solved by sixth order Runge-Kutta-Felberg method. We calculated the transition temperature as a function of amplitude and frequency of oscillating field. This was plotted against field amplitude taking frequency as a parameter. As frequency increases the phase boundary is observed to become inflated. The phase boundary shows an inflection point which separates the nature of the transition. On the dynamic phase boundary a tricritical point (TCP) was found, which separates the nature (continuous/discontinuous) of the dynamic transition across the phase boundary. The inflection point is identified as the TCP and hence a simpler method of determining the position of TCP was found. TCP was observed to shift towards high field for higher frequency. As frequency decreases the dynamic phase boundary is observe to shrink. In the zero frequency limit this boundary shows a tendency to merge to the temperature variation of the coercive field.

Keywords: Ising model, Meanfield theory, Dynamic transition, Tricritical point

I. Introduction:

The ferromagnetic system, in the presence of a time varying external magnetic field, remaining far from statistical equilibrium, became an interesting object of research over the last two decades [1]. One interesting nonequilibrium response is the dynamic phase transition. This dynamic phase transition is widely studied in model ferromagnetic system in the presence of oscillating magnetic field [1]. Tome and Oliveira [2] first observed a prototype of nonequlibrium dynamic transition in the numerical solution of meanfield equation of motion for the classical Ising ferromagnet in the presecce of a magnetic field varying sinusoidally in time. The time averaged (over the complete cycle of the oscillating magnetic field) magnetisation plays the role of the dynamic order parameter. They [2] found that this dynamic ordering depends on the amplitude of the oscillating magnetic field and the temperature of the system. Systems get dynamically ordered for small values of the temperature and the amplitude of the field. They [2] have drawn a phase boundary (separating the ordered and disordered phase) in the temperature field amplitude plane. More interestingly, they have also reported [2] a tricritical point on the phase boundary, which separates the nature (continuous/discontinuous) of the dynamic transition across the phase boundary. This tricritical point was found just by checking the nature of the transition at all points across the phase boundary. The point where the nature of transition changes was marked as the tricritical point. No other significance of this tricritical point was reported. The frequency dependence of this phase boundary was not reported earlier for the dynamic transition in Ising meanfield dynamics.

In this paper, we studied numerically the dynamic transition in Ising meanfield dynamics. Here, we confined our attention to study the frequency dependence of the dynamic phase boundary. We studied the tricritical behaviour and found a method of finding the position the tricritical point on the dynamic phase boundary. The frequency dependence of the position of the tricritical point was studied here. We also studied the static (zero frequency) limit of dynamic phase boundary.

The paper is organised as follows: In the next section the model and the method of numerical solution is discussed. Section III contains the numerical results and the paper end with summary of the work in section IV.

II. Model and numerical solution:

The time (t) variation of average magnetisation m of Ising ferromagnet in the presence of a time varying field, in meanfield approximation, is given as [2]

$$\tau \frac{dm}{dt} = -m + \tanh(\frac{m + h(t)}{T}), \tag{1}$$

where, $h(t)$ is the externally applied sinusoidally oscillating magnetic field ($h(t) = h_0 \sin(\omega t)$) and T is the temperature measured in units of the Boltzmann constant (K_B). This equation describes the nonequilibrium behaviour of instantaneous value of magnetisation $m(t)$ of Ising ferromagnet in mean-field approximation.

We have solved this equation by sixth order Runge-Kutta-Felberg (RKF) [3] method to get the instantaneous value of magnetisation $m(t)$ at any finite temperature T, h_0 and $\omega(= 2\pi f)$. This method of solving ordinary differential equation $\frac{dm}{dt} = F(t, m(t))$, is described briefly as:

$m(t + dt) = m(t) + \left(\frac{16k_1}{135} + \frac{6656k_3}{12825} + \frac{28561k_4}{56430} - \frac{9k_5}{50} + \frac{2k_6}{55}\right)$
where
$k_1 = dt \cdot F(t, m(t))$
$k_2 = dt \cdot F(t + \frac{dt}{4}, m + \frac{k_1}{4})$
$k_3 = dt \cdot F(t + \frac{3dt}{8}, m + \frac{3k_1}{32} + \frac{9k_2}{32})$
$k_4 = dt \cdot F(t + \frac{12dt}{13}, m + \frac{1932k_1}{2197} - \frac{7200k_2}{2197} + \frac{7296k_3}{2197})$
$k_5 = dt \cdot F(t + dt, m + \frac{439k_1}{216} - 8k_2 + \frac{3680k_3}{513} - \frac{845k_4}{4104})$
$k_6 = dt \cdot F(t + \frac{dt}{2}, m - \frac{8k_1}{27} + 2k_2 - \frac{3544k_3}{2565} + \frac{1859k_4}{4104} - \frac{11k_5}{40})$................................(2)

The time interval dt was measured in units of τ (the time taken to flip a single spin). Actually, we have used $dt = 0.01$ (setting τ=1.0). The local error involved in the sixth order RKF method is of the order of $(dt)^6 (= 10^{-12})$. We started with initial condition $m(t = 0) = 1.0$.

III. Results:

The dynamic order parameter $Q(= \frac{2\pi}{\omega} \oint m(t)dt)$ is time average magnetisation over a full cycle of the oscillating magnetic field. This was calculated after discarding the values of Q for few initial (transient [4]) cycles of the oscillating field. Finally, the dynamic order parameter Q is calculated as a

function of T, h_0 and f. Now, depending on the values of these parameters the system gets dynamically ordered ($Q \neq 0$) or disordered ($Q = 0$). This shows a dynamical phase transition which is a nonequilibrium phase transition. We have studied the transition and determined the transition temperature $T_d(h_0, f)$ in a very simple way. For a fixed set of values of h_0 and f the temperature T is varied (in step $\Delta T = 10^{-3}$) and Q is measured as a function of T. Then we calculated the derivative $\frac{dQ}{dT}$ numerically (using three point central difference formula; where the error $O(dT^2)$ [3]). The temperature, at which the derivative $\frac{dQ}{dT}$ is sharply minimum, is considered here as the transition temperature T_d. In this way, we obtained the dynamic transition temperature $T_d(h_0, f)$ for all values of h_0. Here, we have changed h_0 (with interval $\Delta h_0 = 0.02$) and obtained the dynamic transition temperature $T_d(h_0, f)$.

Now, for a particular frequency f, the plot of $T_d(h_0, f)$ against h_0 gives the dynamic phase boundary. This dynamic phase boundary separates the regions of $Q \neq 0$ and $Q = 0$. For fixed frequency, it was observed that the dynamic transition occurs at higher temperature for lower values of applied field amplitude h_0 and vice versa. Fig.1 shows such a variation. For $f = 0.2$ and $h_0 = 0.5$, the temperature variations of Q and $\frac{dQ}{dT}$ are plotted in Fig.1(A) and Fig.1(B) respectively. From the sharp minimum of $\frac{dQ}{dT}$ (in Fig.1(B)) the transition temperature $T_d(h_0, f)$ was found equal to 0.725. The same plots are shown in Fig.1(C) and (D) for $h_0 = 0.3$ (keeping frequency $f = 0.2$ fixed). Here, the transition temperature was found to be equal to 0.919. It is clear from the figure that the transition occurs at lower temperature for higher value of the field amplitude. In this way, the entire phase boundary (i.e., $T_d(h_0, f)$ as a function of h_0 for fixed $f = 0.2$) was obtained.

The dynamic phase boundary was obtained for different frequency f. It was observed that for a fixed value of the field amplitude h_0 that transition occurs at higher temperature for higher frequency. This observation was shown in Fig.2, for fixed $h_0 = 0.4$. The dynamic transition occurs at $T_d(h_0, f) = 0.845$ for $f = 0.2$ (see Fig.2(A) and (B)) and it becomes $T_d(h_0, f) = 0.908$ for $f = 1.0$ (see Fig.2(C) and (D)). From the figures it is clear that the transition occurs at higher temperature for higher frequency. We have reported the results of dynamic phase boundary for frequencies f = 0.01, 0.02, 0.05, 0.1, 0.2, 0.5 and 1.0.

In the limit $f \to 0$ we approach the equilibrium behaviour. In equilibrium,

$\frac{dm}{dt} = 0$. Equation (1) takes the form $m - \tanh\frac{(m+h)}{T} = 0$. This equation was solved by Newton-Raphson iterative method of finding the root to get the value of equilibrium magnetisation $m(h,T)$. At any fixed temperature T, by changing the value of the external magnetic field h, we calculated the coercive field h_c (i.e, the field for which the magnetisation just changes its sign). The value of the coercive field was found to depend on the temperature T, i.e., the coercive field is a function of the temperature.

The dynamic transition temperature T_d is also plotted against the amplitude of the externally applied sinusoidal magnetic field, in the same figure. For a fixed value of frequency, the transition temperature T_d decreases as the value of the field amplitude increases. This gives the dynamic phase boundary, below which we observed the dynamically ordered ($Q \neq 0$) phase and above which the phase is dynamically disordered ($Q = 0$). Different dynamic phase boundary was obtained for different values of frequency and plotted in Fig.3. It is observed that for same value of the field amplitude, the transition temperature T_d increases as the frequency increases. So, the phase boundary gets inflated as the frequency increases.

As the frequency decreases ($f \to 0$) the phase boundary shrinks and ultimately it approches the curve of temperature variation of the coercive field (continuous line in Fig.3). In this context, one may think that the temperature variation of coercive field acts as the static limit of dynamic phase boundary. We studied the nature of the dynamic phase boundary. This dynamic phase boundary changes its curvature from one side to other, as one changes the field from lower to higher value. This means, the boundary has an inflection point (where the curvature changes from one side of the curve to its other side) which was detected by calculating the derivative $\frac{dT_d}{dh_0}$. The derivative $\frac{dT_d}{dh_0}$ plotted against h_0 (for fixed $f = 1.0$) shows (in Fig.4) a very sharp minimum (at h_m) which indicates the inflection point of $T_d - h_0$ curve. This minimum occurs at $h_0 = 0.72$. This minimum or the inflection point has a great significance. If the value of the field amplitude is less than h_m, the transition is a continuous one. A typical transition, for $h_0 = 0.70 (< h_m)$, was shown in the left inset of Fig.4. On the other hand, if the value of the field amplitude exceeds h_m, the transition becomes a discontinuous one. The right inset of Fig.4, shows such a typical transition for $h_0 = 0.74 (> h_m)$. The observation shows that the inflection point on the dynamic phase boundary acts as tricritical point. To search the location of the tricritical point on the dynamic phase boundary, the nature of the

dynamic transition has to be studied at several points. The TCP is the point, where the nature (continuous/discontinuous) of the transition changes from one side to other. But in the method of finding the inflection point on the dynamic phase boundary, one can get the exact location of tricritical point on the phase boundary, quite easily at least in this case. Tome and Oliveira [2] studied this dynamic transition in kinetic Ising model in meanfield approximation and observed the existence of a tricritical point on the phase boundary. But they did not report any method to find the TCP directly from the phase boundary. Here, we found a method of getting the TCP directly from the phase boundary.

We have also studied the change in position of the TCP on the phase boundary as one changes the frequency. Following the same method, by computing the derivative $\frac{dT_d}{dh_0}$ and plotting it against h_0, for different frequency, we obtained the position of the minima of the derivatives (for different frequencies) and hence the position of TCP's. This was shown in Fig.5 for few frequencies. The position of TCP's for different frequencies are shown (by big black dot) in Fig.3. It was observed that the position of TCP shifts towards higher field amplitudes (consequently lower temperatures) for higher frequencies.

IV. Summary:

In this paper, we have reported our numerical results of the study of the dynamic phase transition in kinetic Ising model driven by oscillating magnetic field, in the meanfield approximation. The dynamic phase boundary was drawn in the temperature-field amplitude plane for different frequencies of the applied oscillating magnetic field. The dynamic phase boundary was observed to get inflated as the frequency increases. As the frequency decreases it shrinks and in zero frequency limit it seems to merge to the temperature variation of the coercive field (equilibrium case).

The important thing that we observed here is: The tricritical point on the dynamic phase boundary is the point of inflection of the phase boundary. This observation made the task, of finding the position of TCP on the phase boundary, much simpler. We have also observed that the position of TCP shifts towards higher field amplitude for higher frequency.

Acknowledgments: The library facility provided by Saha Institute of Nuclear Physics, Calcutta, is gratefully acknowledged.

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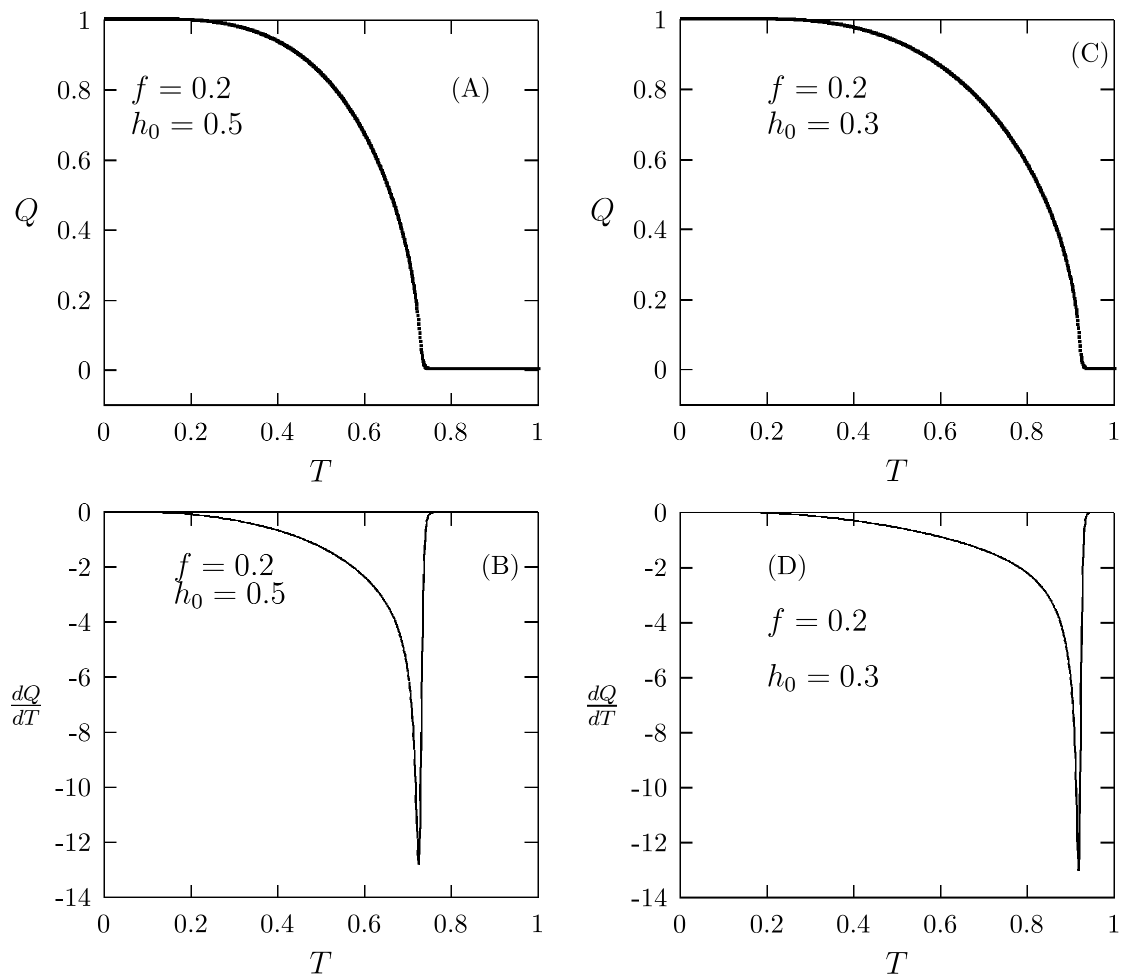

Fig.1. The temperature variation of Q and $\frac{dQ}{dT}$ for different values of h_0 but for fixed $f = 0.2$. (A) and (B) for $h_0 = 0.5$ and (C) and (D) for $h_0 = 0.3$.

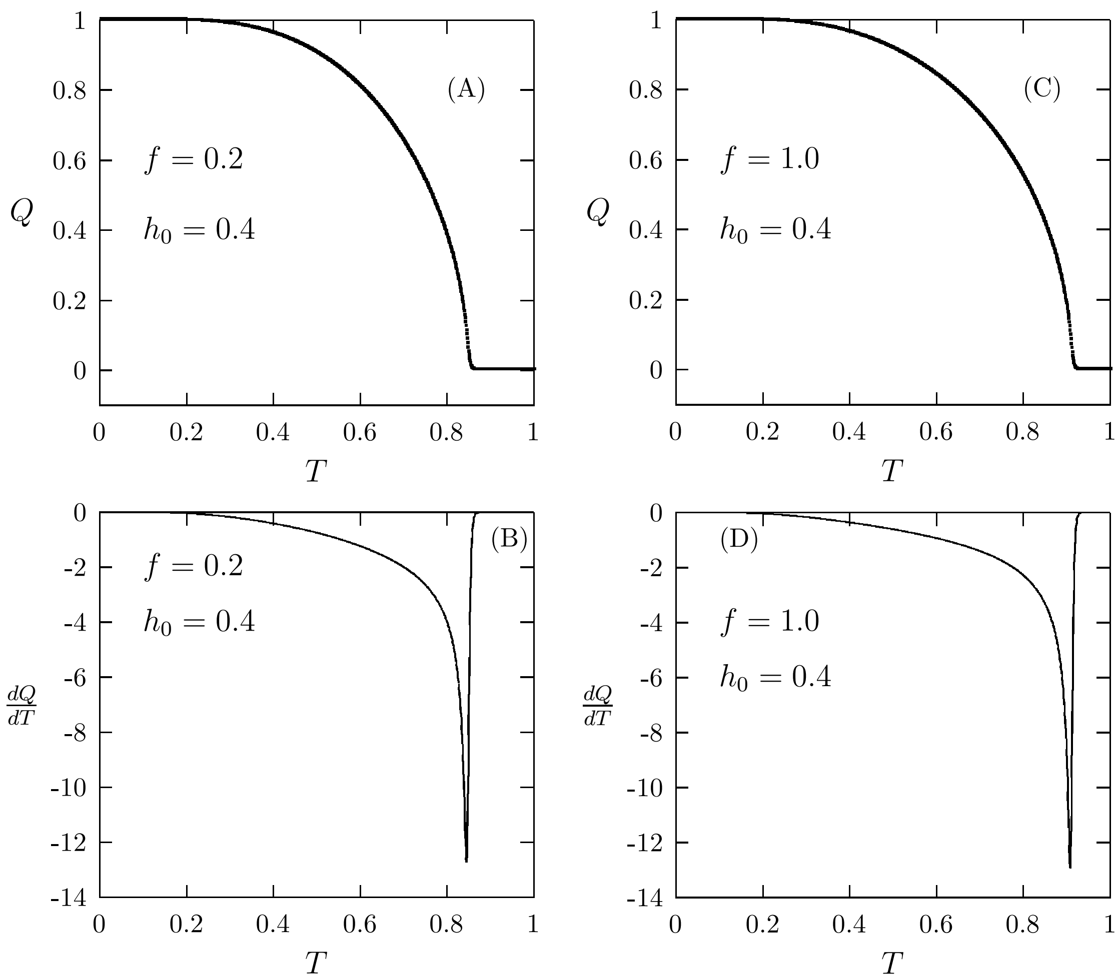

Fig.2. The temperature variation of Q and $\frac{dQ}{dT}$ for different values of f but for fixed $h_0 = 0.4$. (A) and (B) for $f = 0.2$ and (C) and (D) for $f = 1.0$.

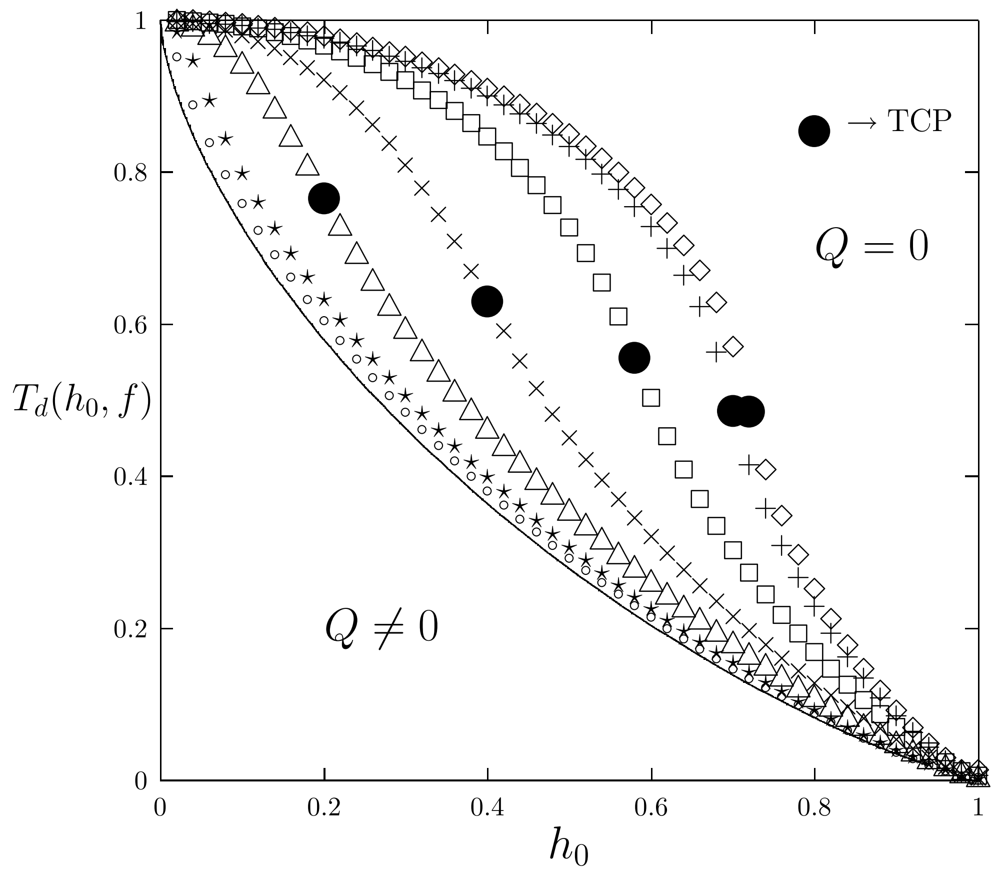

Fig.3. The dynamic transition temperature $T_d(h_0, f)$ is plotted against the amplitude of oscillating magnetic field h_0 taking frequency f as parameter. Different symbols represent different frequencies. $f = 1.0(\diamond)$, $f = 0.5(+)$, $f = 0.2(\Box)$, $f = 0.1(\times)$, $f = 0.05(\triangle)$, $f = 0.02(\star)$ and $f = 0.01(o)$. The bullets represent the TCP's on the phase boundary. The continuous line represent the variation of coercive field with temperature.

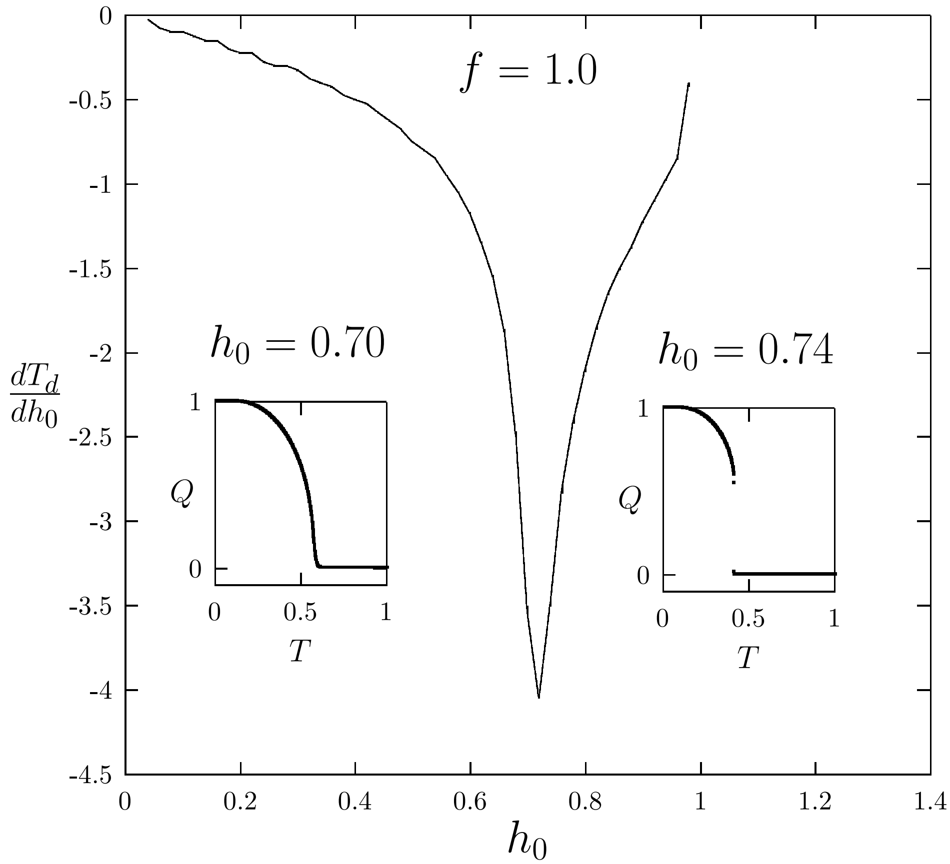

Fig.4. The derivative $\frac{dT_d}{dh_0}$ is plotted against the field amplitude h_0 for a particular frequency $f = 1.0$. Left inset shows the temperature variation of the dynamic order parameter for $h_0 = 0.70$ and the right inset shows that for $h_0 = 0.74$.

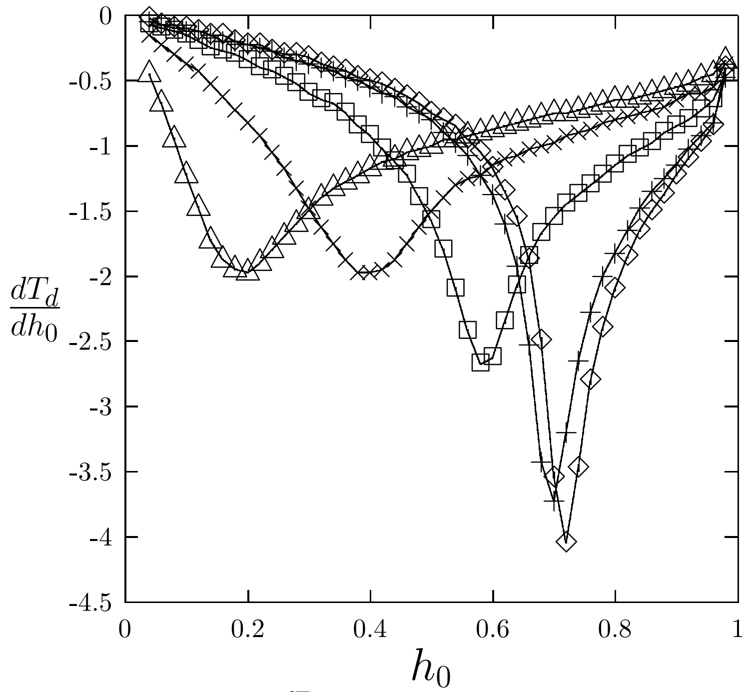

Fig.5. The derivative $\frac{dT_d}{dh_0}$ is plotted against the field amplitude h_0 for different frequencies. Different symbols represent different frequencies. $f = 1.0(\diamond)$, $f = 0.5(+)$, $f = 0.2(\Box)$, $f = 0.1(\times)$ and $f = 0.05(\triangle)$. The continuous line in each case just connects the data points.